Instructions: Complete each of the following exercises for practice.

1. Verify that the Divergence Theorem holds by computing $\iiint_R \operatorname{div}(\mathbf{F})\, dV$ and $\iint_{\partial S} \mathbf{F} \cdot d\mathbf{S}$ separately.
   (a) $\mathbf{F}(x, y, z) = \langle 3x, xy, 2xz\rangle$, R the cube bounded by the planes $x = 0$, $y = 0$, $z = 0$, $x = 1$, $y = 1$, and $z = 1$
   (b) $\mathbf{F}(x, y, z) = \langle y^2z^3, 2yz, 4z^2\rangle$, R the solid enclosed by $z = x^2 + y^2$ and $z = 9$
   (c) $\mathbf{F}(x, y, z) = \langle z, y, x\rangle$, R the solid ball $x^2 + y^2 + z^2 \leq 16$
   (d) $\mathbf{F}(x, y, z) = \langle x^2, -y, z\rangle$, R the solid cylinder $y^2 + z^2 \leq 9$, $0 \leq x \leq 2$

2. Use the Divergence Theorem to compute the flux of $\mathbf{F}$ across S.
   (a) $\mathbf{F}(x, y, z) = \langle xye^z, xy^2z^3, -ye^z\rangle$, S the surface of the box bounded by the coordinate planes and $x = 3$, $y = 2$, and $z = 1$
   (b) $\mathbf{F}(x, y, z) = \langle x^2yz, xy^2z, xyz^2\rangle$, S the surface of the box bounded by the coordinate planes and $x = a$, $y = b$, and $z = c$ for $a, b, c > 0$
   (c) $\mathbf{F}(x, y, z) = \langle 3xy^2, xe^z, z^3\rangle$, S the boundary of the solid cylinder $y^2 + z^2 \leq 1$, $-1 \leq x \leq 2$
   (d) $\mathbf{F}(x, y, z) = \langle x^3 + y^3, y^3 + z^3, z^3 + x^3\rangle$, S the sphere of radius 2 centered at the origin
   (e) $\mathbf{F}(x, y, z) = \langle xe^y, z - e^y, -xy\rangle$, S the ellipsoid $x^2 + 2y^2 + 3z^2 = 4$
   (f) $\mathbf{F}(x, y, z) = \langle z, y, zx\rangle$, S the boundary of the tetrahedron enclosed by the coordinate planes and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ where $a, b, c > 0$
   (g) $\mathbf{F}(x, y, z) = \langle 2x^3 + y^3, y^3 + z^3, 3y^2z\rangle$, S the boundary of the region enclosed by paraboloid $z = 1 - x^2 - y^2$ and the xy-plane
   (h) $\mathbf{F}(x, y, z) = \langle xy + 2xz, x^2 + y^2, xy - z^2\rangle$, S the boundary of the region in the cylinder $x^2 + y^2 = 4$ and between planes $z = y - 2$ and $z = 0$

3. Suppose $R \subseteq \mathbb{R}^3$ satisfies the conditions of the divergence Theorem, $\mathbf{C}$ is a constant vector, and assume vector field $\mathbf{F}$ and scalar fields f and g have continuous second-order partial derivatives on an open region containing R. Prove each of the following.
   (a) $\iint_{\partial R} \mathbf{C} \cdot d\mathbf{S} = 0$
   (b) $Vol(R) = \frac{1}{3} \iint_{\partial R} \langle x, y, z\rangle \cdot d\mathbf{S}$
   (c) $\iint_{\partial R} \operatorname{curl}(\mathbf{F}) \cdot d\mathbf{S} = 0$
   (d) $\iint_{\partial R} D_{\mathbf{n}} f\; dS = \iiint_R \nabla^2 f\; dV$
   (e) $\iint_{\partial R} f\nabla g \cdot d\mathbf{S} = \iiint_R (f\nabla^2 g + \nabla f \cdot \nabla g)\; dV$
   (f) $\iint_{\partial R} (f\nabla g - g\nabla f) \cdot d\mathbf{S} = \iiint_R (f\nabla^2 g - g\nabla^2 f)\; dV$